

OPM 662 – Business Analytics: Modeling and Optimization

(Assignment 4)

By

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# **Problem Structure**

An operation room undertakes about 42% of a hospital revenue, so as to maximize the profits of operation, it need to reduce the downtime of both ORs and surgeons as low as possible. Due to many different constraints such as limited number of surgeons and uncertainty of the time required to perform OR procedures, optimization task of ORs operation at hospital is required to be carried out.

## Inputs

* Surgeries to be carried out
* Preparation time for each kind of surgeries
* Surgery time for each kind of surgeries
* Clean-up time for each kind of surgeries
* Normal shift
* Cost per hours of having an OR vacant
* Cost per hours of having the surgeon waiting
* Cost per hours of using OR stuff in any of the ORs beyond their normal time shift

## Decisions

* The best sequence of OR assignment for each of the surguries

## Objective

The objective of the problem is to minimize the total vacant and overtime costs of the ORs and the total waiting time cost of surgeon.

## Constraints

* Single surgeon constraint:single surgeon might go from one OR to the other after completing the surgery time

# **Optimization Model**

## Algebraic Formulation

### **2.1.1 Indices**

Surgeries

Operation Rooms

### **2.1.2 Parameters**

Preparation time for surgery i

Surgery time(or incision time) for surgery i

Clean-up time for surgery i

Cost of vacant room

Cost of waiting time

Cost of overtime

A large enough number

### **2.1.3 Decision Variables**

The starting time of incision of surgery

The time at which surgeon finishes last surgery

Surgeon overall waiting time within the whole shift

Equal to 1 if surgery is performed directly or indirectly infront of surgery ,otherwise equal to 0

Equal to 1 if surgery is performed at room

The time at which OR finishes last clean-up

OR ’s total vacant time within the whole shift

OR staff’s overtime in OR

OR ’s unused shift time

Starting time of surgeon in her first shift

### **2.1.4 Objective Function**

### **2.1.5 Constraints**

(1)

(2)

(3)

(4)

(5)

(6)

*-1*  (7)

(8)

(9)

*+*

(10)

(11)

## Explanations

### **2.2.1 Objective Function**

The objective of the model is to minimize the sum of overtime cost per room ,and vacant time cost per room as well as the waiting time cost for surgeon.

### **2.2.2 Earliest starting time constraints for surgeon**

Constraint (1) is the earliest starting time constraints for surgeon. We let the starting time of surgery i larger than it’s preparing time to ensure that the first performed surgery starts only after it’s preparing time.

### **2.2.3 Surgeon working time constraints**

Constraints (2)(3)(4) together form the constraints for the surgeon working time.

* Constraint (2) ensures that the starting time of surgeon is smaller than the starting time of any surgery incision time ,so we can get the first time span between T=0 and the starting time of surgeon’s first incision given our minimizing objective function.
* Constraint (3) ensures that surgeon make span is larger than any surgery’s incision starting time plus it’s surgery time, so we can know the time when surgeon finishes all his surgery.
* Constraint (4) is to calculate the waiting time of surgeon. Waiting time should equal to the total time before surgeon leaves OR minus the sum of all surgeries incision time minus the starting time of surgeon’s first surgery given that the waiting time is only considered for time span between two following incision.

### **2.2.4 No surgeon** **overlapping constraints**

Constraints (5)(6) together form the constraints for the non-overlapping constraints.

* Constraint (5) ensures that if surgery i is scheduled indirectly or directly before surgery j,the starting incision time of surgery j should smaller than the starting incision time of surgery i plus surgery i’s surgery time. Surgeon can only go from one surgery to another after completing the incision time in a surgery.
* Constraint (6) ensures that surgery i can only be scheduled before or after surgery j. There are no other choices.

### **2.2.5 Surgery schedule constraints**

Constraints (7)(8)(9)(10)(11) together form the constraints for the surgery constraints.

* Constraint (7) is to get the time for each OR when the last surgery clean-up is finished. For room r, if surgery i is scheduled in room r, ORSM should larger than starting incision time, incision time plus clean-up time of any surgery i .
* Constraint (8) ensures that for OR r, the time for each OR when the last surgery clean-up is finished,should be equal to normal shift time plus overtime if working time of OR is larger than normal shift time,so that we can obtain overtime for each room for calculate overtime cost.If working time of OR ,is smaller than normal shift time,we can also attain unused normal shift time .
* Constraint (9) is to calculate the vacant time for each room . The total vacant time of OR r equals to the remaining time after deducting the total preparation, incision and clean-up time of surgery i in OR r.
* Constraints(10) is to constraint no overlapping for surgery with time sequence in same operation room. The incision of surgery j can only begin after finishing surgery i in OR r if both surgeries are scheduled to take place in OR r and if surgery i is directly or indirectly in front of surgery j.
* Constraints(11) is to make sure that each surgery i can only take place in one OR.

# **Results**

## Instance 1

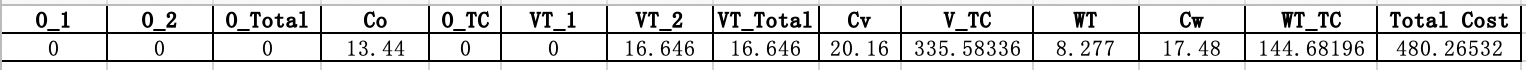
After the model is resolved in Python, the following solution can be obtained. In instance 1,surgery A, A,C,J need to be scheduled. The optimal objective value is 480.265. The question about corresponding assigned operation room and sequence of surgeries, and overtime cost ,vacant time cost as well as surgeon waiting cost are showed in the table below. In our optimal solution, firstly, the surgeon performed surgery A in OR1, then went to OR 2 for surgery J, then went to OR 1 for surgery C, and finally went to OR 2 for surgery A. Total overtime cost is equal to 0. Total vacant room cost is equal to 335.58. Total surgeon’s waiting cost is equal to 144.68.

Table 1: Schedule of Instance 1

图表

描述已自动生成

Table 2: The cost in Instance 1



## Instance 2

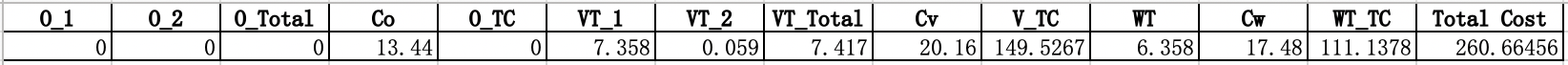
In instance 2,surgery A,D,G,G,J need to be scheduled. The optimal objective value is 260.66. The question about corresponding assigned operation room and sequence of surgeries, and overtime cost ,vacant time cost as well as surgeon waiting cost are showed in the table below. In our optimal solution, firstly, the surgeon performed surgery A in OR 2, then went to OR 1 for surgery D, then went to OR 2 for surgery G, then went to OR 1 for surgery G ,and finally went to OR 2 for surgery J. Total overtime cost is equal to 0. Total vacant room cost is equal to 149.53. Total surgeon’s waiting cost is equal to 111.14.

Table 3: Schedule in Instance 2

图表, 条形图

描述已自动生成

Table 4: The cost in Instance 2



## Instance 3

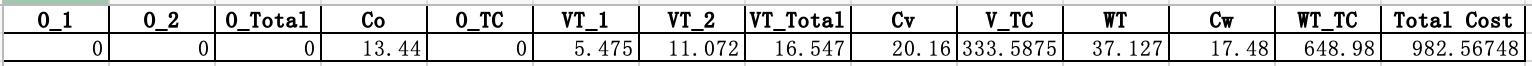
In instance 3,surgery A,B,D,E,G,G,J need to be scheduled. The optimal objective value is 982.57. The question about corresponding assigned operation room and sequence of surgeries, and overtime cost ,vacant time cost as well as surgeon waiting cost are showed in the table below. In our optimal solution, firstly, the surgeon performed surgery A in OR 2, then went to OR 1 for surgery J, then went to OR 2 for surgery G, then went to OR 1 for surgery D ,then went to OR 2 for surgery G, then went to OR 1 for surgery B and finally went to OR 2 for surgery E. Total overtime cost is equal to 0. Total vacant room cost is equal to 333.59. Total surgeon’s waiting cost is equal to 648.98.

Table 5: Schedule in Instance 3

图表, 条形图

描述已自动生成

Table 6: The cost in Instance 3



## Instance 4

In instance 4,surgery A,A,C,E,E,F,G,,H,I,I,J need to be scheduled. The optimal objective value is 4363.87. The question about corresponding assigned operation room and sequence of surgeries, and overtime cost ,vacant time cost as well as surgeon waiting cost are showed in the table below.The order of surgery is showed in Table 9. Total overtime cost is equal to 1989.33. Total vacant room cost is equal to 348.73. Total surgeon’s waiting cost is equal to 2155.81.

Table 7: Schedule in Instance 4

图表

描述已自动生成

Table 8: The cost in Instance 4

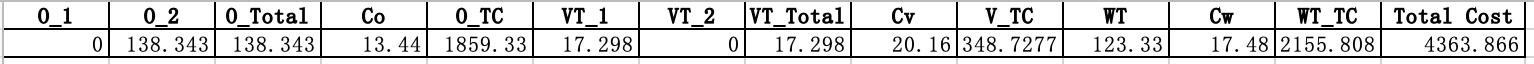


Table 9: The surgery sequence in Instance 4

表格

描述已自动生成

## Conclusion

The corresponding cost of 4 instances is showed in following chart.We can see that except for instance 4,the total cost of all other instance is lower than 1000.This is because the increase in the number of surgery leads to surge in overtime of OR staff.Besides,the limited number of OR also makes surgeon to waiting so long, which raises the total cost further. To reduce cost ,it is necessary to control the number of daily surgery or increase the number

图表, 条形图

描述已自动生成

# **Stochastic Model**

## Algebraic Formulation

### **2.1.1 Indices**

Surgeries

Operation Rooms

Scenarios

### **2.1.2 Parameters**

Preparation time for surgery i in scenarios s

Surgery time(or incision time) for surgery i in scenarios s

Clean-up time for surgery i in scenarios s

Cost of vacant room

Cost of waiting time

Cost of overtime

A large enough number

### **2.1.3 Decision Variables**

The starting time of incision of surgery

The time at which surgeon finishes last surgery

Surgeon overall waiting time within the whole shift

Equal to 1 if surgery is performed directly or indirectly infront of surgery ,otherwise equal to 0

Equal to 1 if surgery is performed at room

The time at which OR finishes last clean-up

OR ’s total vacant time within the whole shift

OR staff’s overtime in OR

OR ’s unused shift time

Starting time of surgeon in her first shift

### **2.1.4 Objective Function**

### **2.1.5 Constraints**

(1)

(2)

(3)

(4)

(5)

(6)

*-1*  (7)

(8)

(9)

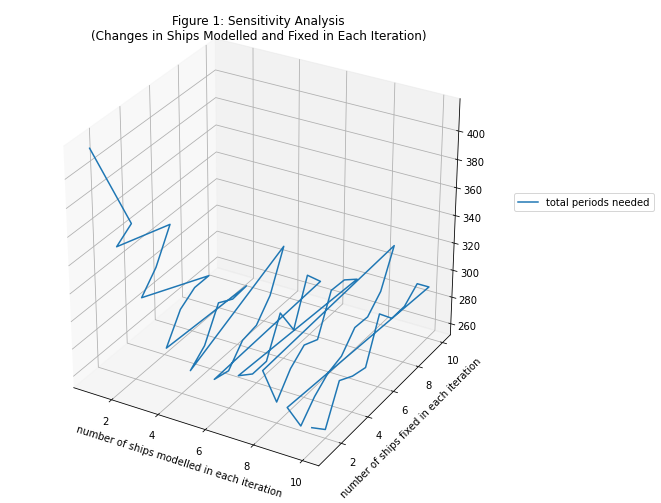
*+*

(10)

(11)

# **Decomposition Heuristic Approach**

## Sensitivity analysis for parameters

For the heuristic solution approach, decomposition approach is chosen instead of aggregation approach. This is because aggregation approach is not suitable for our case as neither product which is ship in this case nor time can be aggregated and solved altogether. Each ship needs to be resolved individually as their order or unloading time cannot be combined with the other ships. So is the case for time periods. Therefore, decomposition approach is selected which decomposes the model into smaller models to be run in multiple iterations and fixes some of the variables determined in each iteration in the next iteration. Within the approach, it is decided that overlapping format is implemented instead of independent format as overlapping format can allow for modelling multiple ships at once while fixing only some of the ships in that iteration which is anticipated to yield a better optimization result than that of modelling each ship independently in each iteration. However, the parameters including the number of ships to be modelled in each iteration and the number of ships to be fixed in each iteration are crucial in obtaining the optimization result from the approach, hence, a sensitivity analysis is carried out to find out which combination of the parameters will yield a better outcome. Nevertheless, the purpose of the approach is to reduce the complexity of the problem, it does not make sense to be putting in high numbers into the parameters as that might only increase the complexity of the problem. Therefore, we think it might make more sense to limit the range to 10 ships as the maximum as like in instance 2 since we have tried higher numbers which take a relatively long time to implement. In the end, according to the result of sensitivity analysis which is shown in the graph and table below, it is the most optimal to set the number of ships to be modelled in each iteration to be nine and the number of ships to be fixed in each iteration to be two. It can also be observed that the objective value of modelling each ship independently in each iteration to be the highest among all the other solutions which proves our previous expectation to be correct.

## Result from decomposition approach

By setting the parameters to be nine and two, total number of periods (objective value) for all the 20 ships to pass through the jetty is 255 periods. Once again, the order of the ships and the final result of the model are shown in the tables below. Once again, same as the above scenarios, most of the ships are scheduled to unload at the stations with the least unloading time. However, the situation that some ships stay in the station even longer than its unloading time or one period pass-through time has increased, such as for ship Duandra and ship Hansa. This can be due to the increasing size and complexity of the model. On the other hand, according to the CPU time used shown in Python, it only takes around 0.062 seconds for running the model which seems quite efficient. Although it might be possible to use parameters with higher numbers which might derive a better solution, the tradeoff between the CPU time and performance might not worth it by raising the complexity.

Table 5: Entering Order of Ships in Instance 3

Table

Description automatically generated

Table 6: The Final Schedule of Ships in Instance 3

# **Conclusion**